

A romantic walk under rain and partial derivatives

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A personal story about discovering the beauty of partial derivatives through observing rain patterns during long commutes to high school. This post explores how mathematical concepts like gradients and partial derivatives can elegantly describe the changing intensity of rain as you move through space and time, culminating in a romantic walk where these mathematical insights were shared with a loved one.



Figure 1: Couple walking under rain. One is thinking about partial derivatives. Credit: [Chris McMorrow](#)

Long distance commute

My high school was about 50 km (~30 miles) away from where I lived. The public transport was painfully slow, and I had to take two or three buses, hence it took 1.5 hours on the average, sometimes way longer than that, to get to the school. I would either read a book, or just sleep. One thing that fascinated me every single time was the weather, the rain in particular. I would get on the bus on a sunny morning with clear skies, only to witness heavy rain in the middle of my trip, and still arrive to school under the sun. Was the rain starting and ending all across the city, or was it showering here and there on my path from home to school? I figured it was possibly a combination. However, I had no way of describing it quantitatively since such questions couldn't be addressed with the level of math we were learning at school.

And, I wasn't even good at math! That stayed as an open question in my mind for quite some time.

The Math

All the physics and sci-fi books I read during my long commutes led me to major in physics in college. And it wasn't too long before I learned the basic math concepts to address the problem of changing rain patterns. It was essentially derivatives, the partial ones, though. Consider the image below:

This is a static copy. Find the animation [here](#).

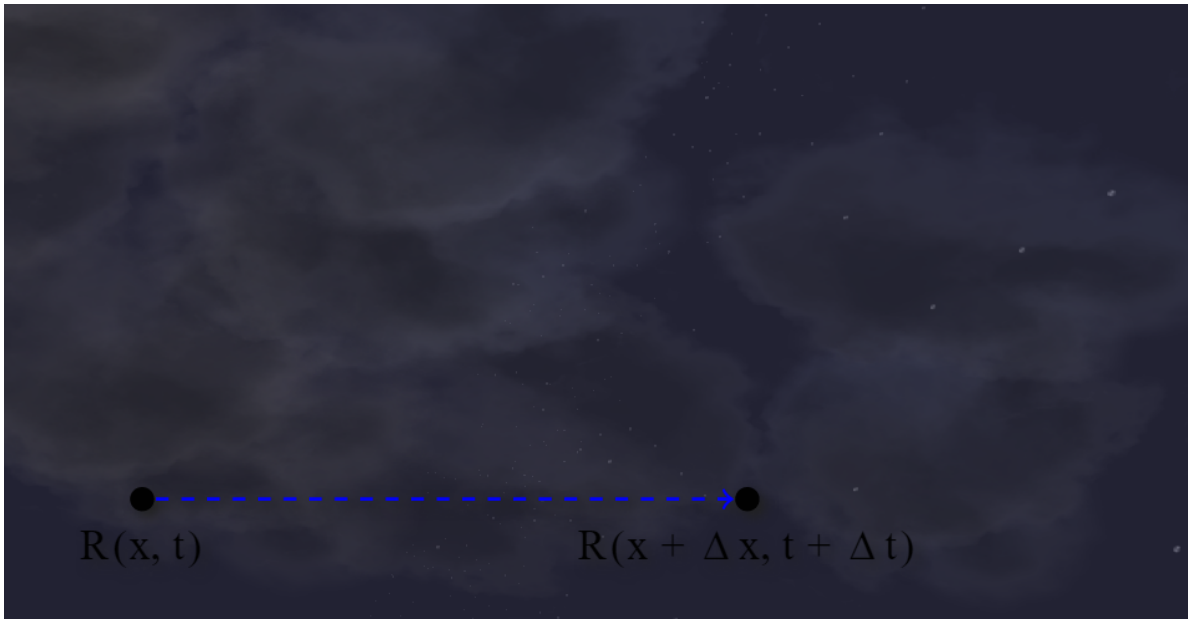


Figure 2: An animation with three.js. It is only raining on the right hand side

You start from a point x at time t at which the intensity of the rain is denoted as $R(x, t)$. Then, you travel to a new point $x + \Delta x$, and the amount of time it takes you to get there is Δt . Since you have moved in space-time, the rain intensity may have changed because it has taken you some time to get there, and/or it was already at a different rate in that second location. Let's define the change in the rain intensity as follows:

$$\Delta R \equiv R(x + \Delta x, t + \Delta t) - R(x, t) \quad (1)$$

$$= R(x, t) + \frac{\partial}{\partial x} R(x, t) \Delta x + \frac{\partial}{\partial t} R(x, t) \Delta t - R(x, t) + \text{H.O.T} \quad (2)$$

$$\simeq \frac{\partial}{\partial x} R(x, t) \Delta x + \frac{\partial}{\partial t} R(x, t) \Delta t, \quad (3)$$

where we expanded $R(x + \Delta x, t + \Delta t)$ at the first order. Let us now take a look at the rate of change in the rain intensity by dividing Eq. 3 by Δt to get:

$$\frac{\Delta R}{\Delta t} \equiv \frac{dR}{dt} = \frac{\partial}{\partial x} R(x, t) \frac{\Delta x}{\Delta t} + \frac{\partial}{\partial t} R(x, t) \quad (4)$$

$$= \frac{\partial}{\partial x} R(x, t) v + \frac{\partial}{\partial t} R(x, t) \quad (5)$$

where we defined $v = \frac{\Delta x}{\Delta t}$, which is nothing but the velocity you are travelling at. Eq. 5 clearly shows the total change in the rain intensity, i.e., $\frac{dR}{dt}$ has two components:

1. The explicit change in time $\frac{\partial R(x, t)}{\partial t}$
2. The implicit change due to change of location: $\frac{\partial R(x, t)}{\partial x}$

Since we have come so far, let's seal the deal by looking at a case where you don't just travel along the x -axis but, along any arbitrary axis, i.e., moving from (x, y, z) to $(x + \Delta x, y + \Delta y, z + \Delta z)$. In this case, the change in the rain intensity is:

$$\frac{\Delta R}{\Delta t} \equiv [R(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t) - R(x, y, z, t)] / \Delta t \quad (6)$$

$$\simeq \frac{\partial R}{\partial x} \frac{\Delta x}{\Delta t} + \frac{\partial R}{\partial y} \frac{\Delta y}{\Delta t} + \frac{\partial R}{\partial z} \frac{\Delta z}{\Delta t} + \frac{\partial R}{\partial t} \quad (7)$$

$$= \frac{\partial R}{\partial x} v_x + \frac{\partial R}{\partial y} v_y + \frac{\partial R}{\partial z} v_z + \frac{\partial R}{\partial t}, \quad (8)$$

where $v_{x,y,z}$ shows the velocities.

It becomes very economical to define the velocity vector $\vec{v} = (v_x, v_y, v_z)$ and the partial derivative vector $\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$, and re-write Eq. 8 as

$$\frac{dR}{dt} \equiv \vec{v} \cdot \vec{\nabla} R + \frac{\partial R}{\partial t}, \quad (9)$$

which beautifully summarizes everything. If the direction of the velocity (\vec{v}) is aligned with the direction the rain intensity changes the most, i.e., $\vec{\nabla}R$, you will observe the highest amount of change due to your movement. On top of it, the rain intensity may change with time independent of your movement.

The Walk

Once I figured out that it was all about the partial derivatives, I just couldn't help thinking about it whenever I had to walk or travel under rain, no matter what the circumstances are. One of those instances happened to be a romantic walk with my then-girlfriend. And I decided that it was a good topic to talk about on a date. She did find it interesting, when I first mentioned it. Probably, I shouldn't have brought it up every single time!

However, exactly this day, 14 years ago, she has committed herself to listening to my partial derivative blurb every time we happen to walk under rain.

Happy anniversary, love you!

References

- threejs. (2020). JavaScript 3D library. <https://github.com/mrdoob/three.js/>
- Red Stapler. (2019). Three.js Realistic Rain Effect Tutorial. <https://redstapler.co/three-js-realistic-rain-tutorial/>