Real coils are not purely imaginary

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Real coils deviate from idealized models due to finite wire resistance, skin depth effects, and parasitic capacitance. We analyze manufacturer data to extract key parameters and develop predictive models for practical coil behavior. This analysis is essential for accurate circuit optimization, particularly in applications such as LIGO RF filter design.

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Practical coils

In order to build a predictive model, it is essential to include various aspects of real coils which deviate from idealized coils. First of all, it is wound with wires of finite resistance, and therefore there is a resistive component to it. Wires stacked together also cause a capacitive effect. Overall, a practical coil can be modeled as shown in Figure 1.

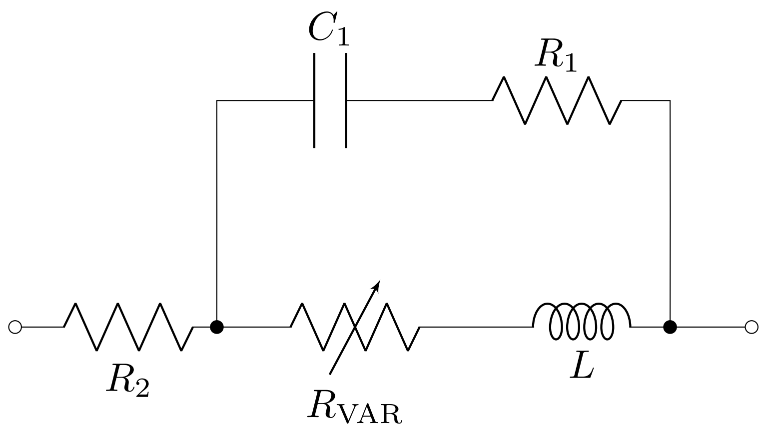


Figure 1: The equivalent circuit for a real coil. In addition to the inductance, we have serial resistance originating from finite wire conductivity, skin depth effect and parallel capacitance.

R_{VAR} in Figure 1 represents the skin effect, and it is given by

$$R_{\mathrm{VAR}} = k\sqrt{f}, \tag{1}$$

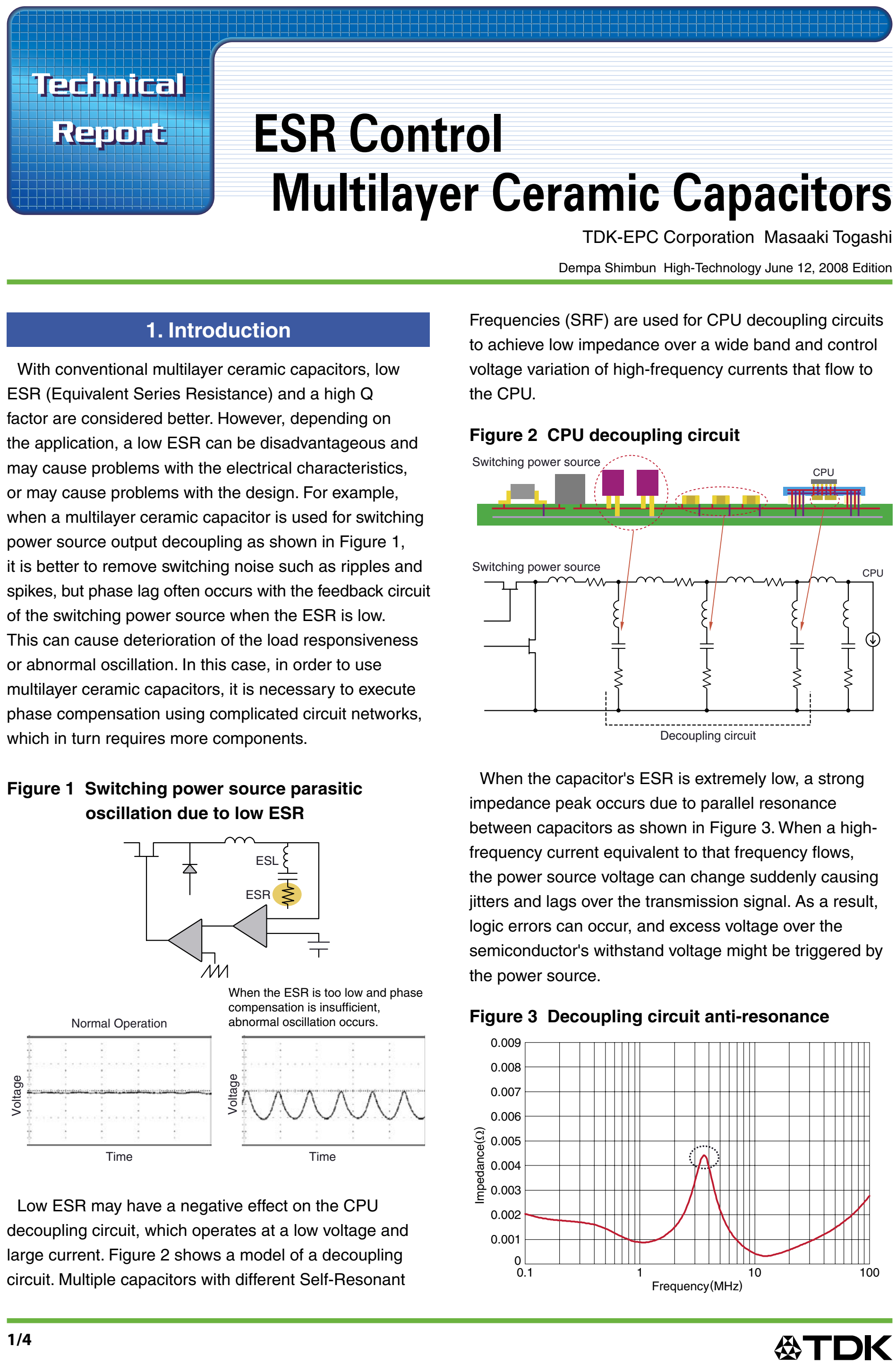
where k is a constant we will extract from manufacturer data. We also analyze CoilCraft's product data sheet, remove suspected data points and use regression to estimate the parameters of the practical coils that we use in the optimization process. The most significant deviation from an ideal coil is the serial resistance, R_2, and we can set our expectations about it without even looking at the data. The inductance of a coil is proportional to the square of the number of turns. This simply follows from the fact that the field created by a each turn passes through every other turn as well, i.e., the interaction is $\propto N(N-1) \sim N^2$. The length of the wire used in the coil is proportional to the number of turns at the first order (a small quadratic order term appears as the turn diameter increases with more turns.) Overall we can expect:

$$R_2 \propto N \propto \sqrt{L}. \tag{2}$$

We can also argue that k is linear function of the magnetic field, which inturn proportional to the inductance. Therefore, if I was forced to make a prediction, I would predict the following relation:

$$k \propto L. \tag{3}$$

Manufacturers tabulate and publish these parameters. We will use the data from CoilCraft, as shown in Figure 2.



Technical Report

ESR Control Multilayer Ceramic Capacitors

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1. Introduction

With conventional multilayer ceramic capacitors, low ESR (Equivalent Series Resistance) and a high Q factor are considered better. However, depending on the application, a low ESR can be disadvantageous and may cause problems with the electrical characteristics, or may cause problems with the design. For example, when a multilayer ceramic capacitor is used for switching power source output decoupling as shown in Figure 1, it is better to remove switching noise such as ripples and spikes, but phase lag often occurs with the feedback circuit of the switching power source when the ESR is low. This can cause deterioration of the load responsiveness or abnormal oscillation. In this case, in order to use multilayer ceramic capacitors, it is necessary to execute phase compensation using complicated circuit networks, which in turn requires more components.

Figure 1 Switching power source parasitic oscillation due to low ESR

Low ESR may have a negative effect on the CPU decoupling circuit, which operates at a low voltage and large current. Figure 2 shows a model of a decoupling circuit. Multiple capacitors with different Self-Resonant Frequencies (SRF) are used for CPU decoupling circuits to achieve low impedance over a wide band and control voltage variation of high-frequency currents that flow to the CPU.

Figure 2 CPU decoupling circuit

When the capacitor's ESR is extremely low, a strong impedance peak occurs due to parallel resonance between capacitors as shown in Figure 3. When a high-frequency current equivalent to that frequency flows, the power source voltage can change suddenly causing jitters and lags over the transmission signal. As a result, logic errors can occur, and excess voltage over the semiconductor's withstand voltage might be triggered by the power source.

Figure 3 Decoupling circuit anti-resonance

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Figure 2: A leading coil manufacturer has a document on their coil model parameters.

Table 1: Coilcraft inductor parameters for various products.

Let's look for some trends and outliers.

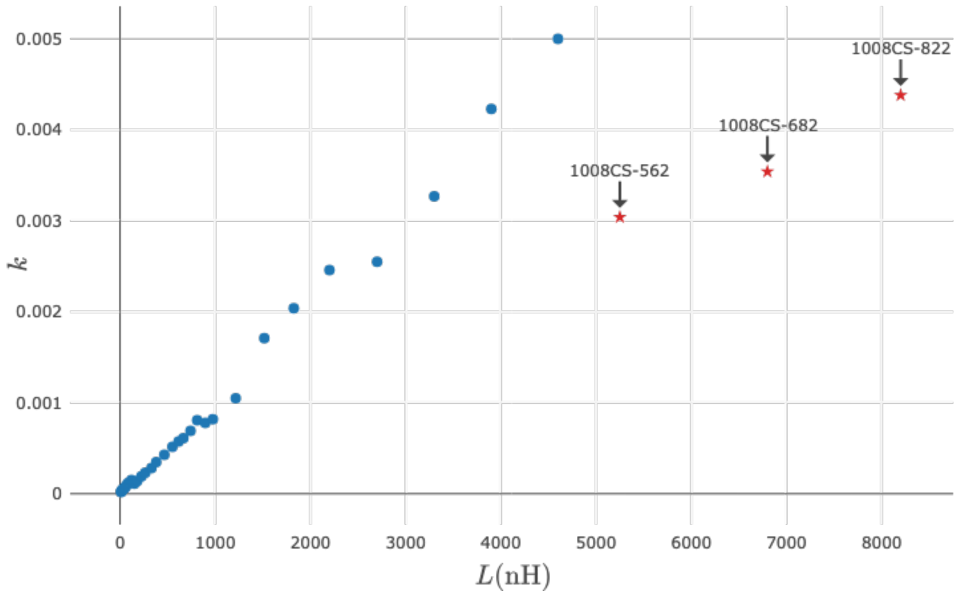

Figure 3: The parameter k vs L.

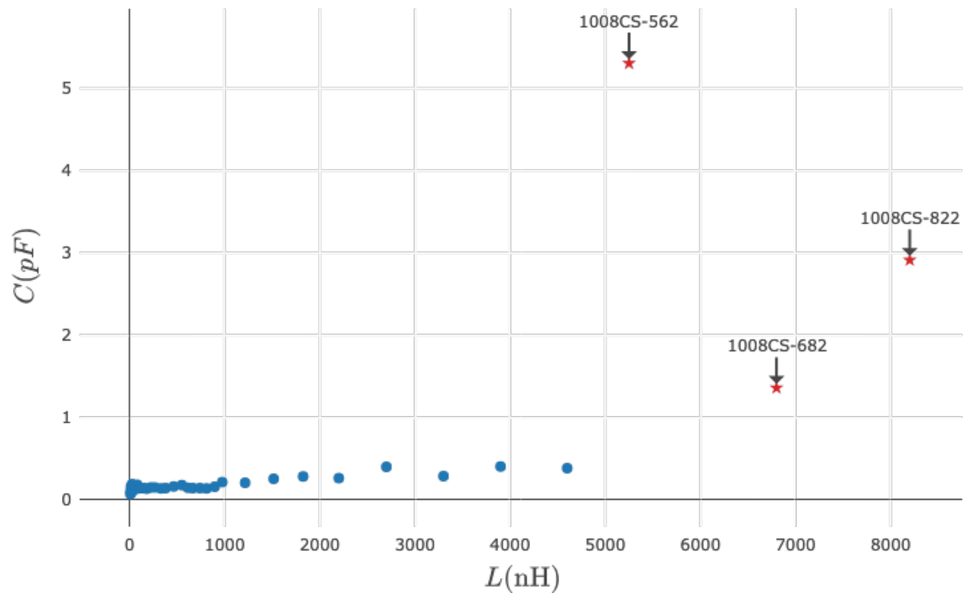

Figure 4: C vs L.

After removing the outlier points, we can check if the predictions in Eqs. 2 and 3 hold by doing a scatter plot and a linear fit as seen in Figure 5.

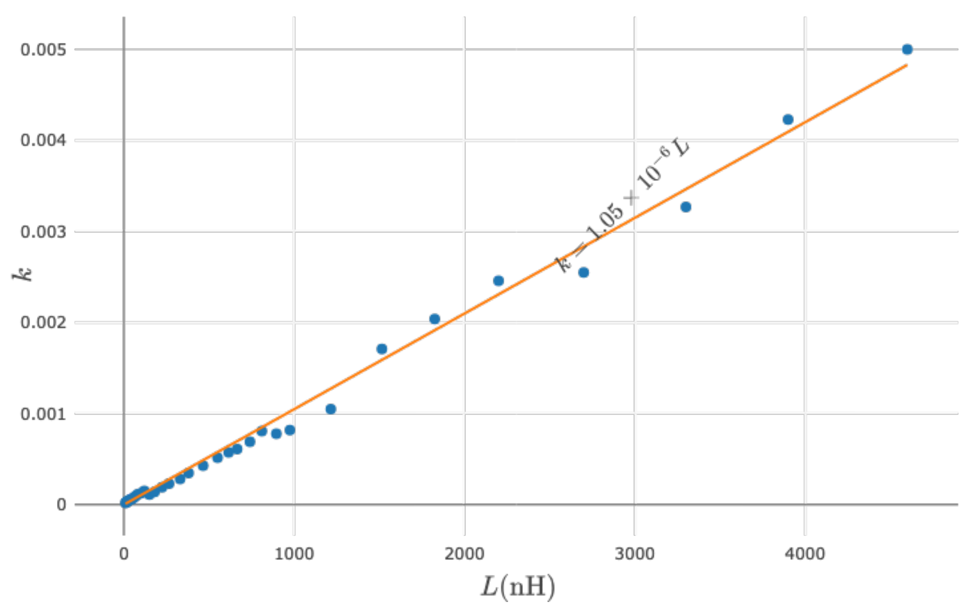

Figure 5: Linear fit to the parameter k vs L.

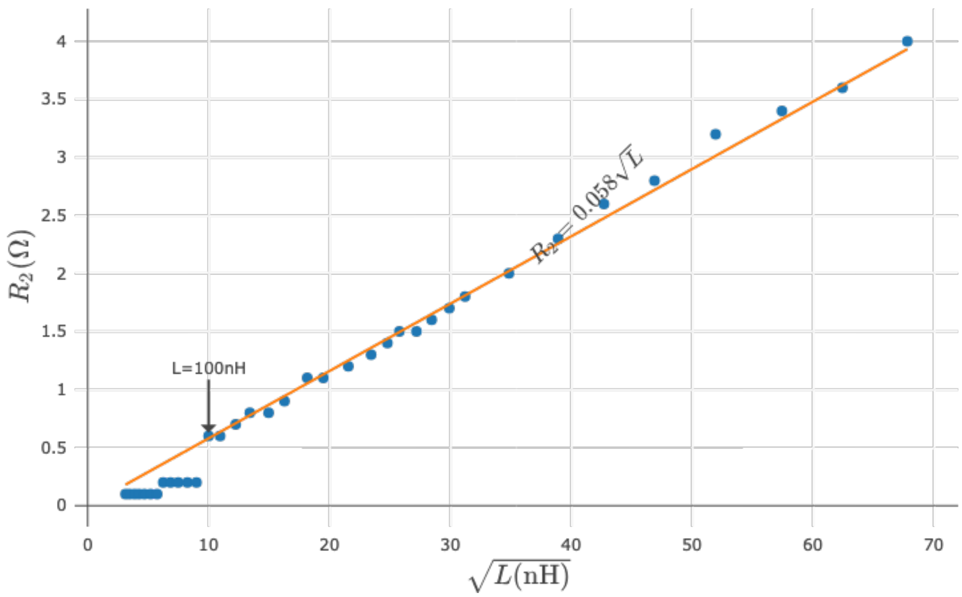

Figure 6: Linear fit to the parameter R_2 vs $\sqrt{L}$.

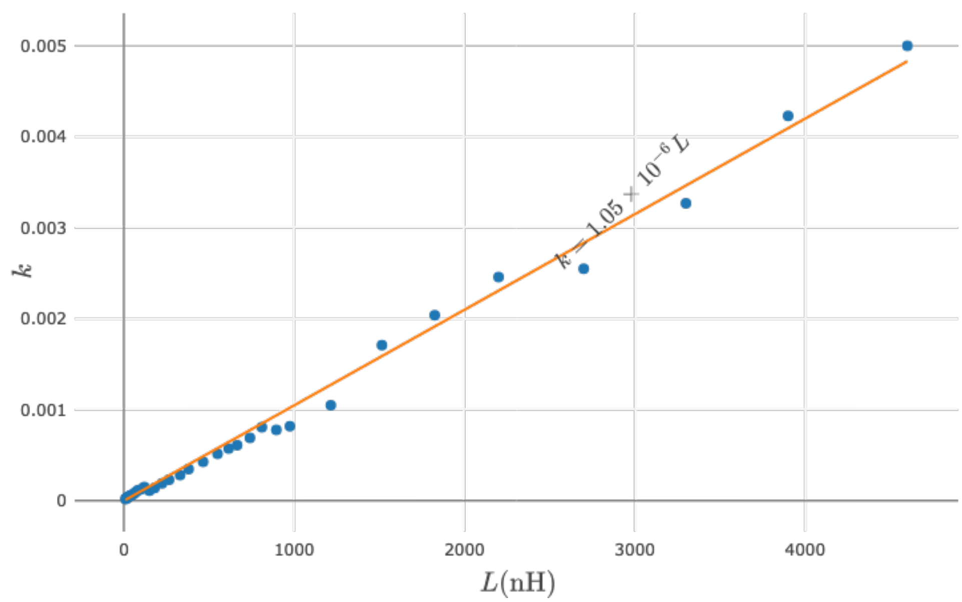

Figure 7: Linear fit to the parameter k vs L.

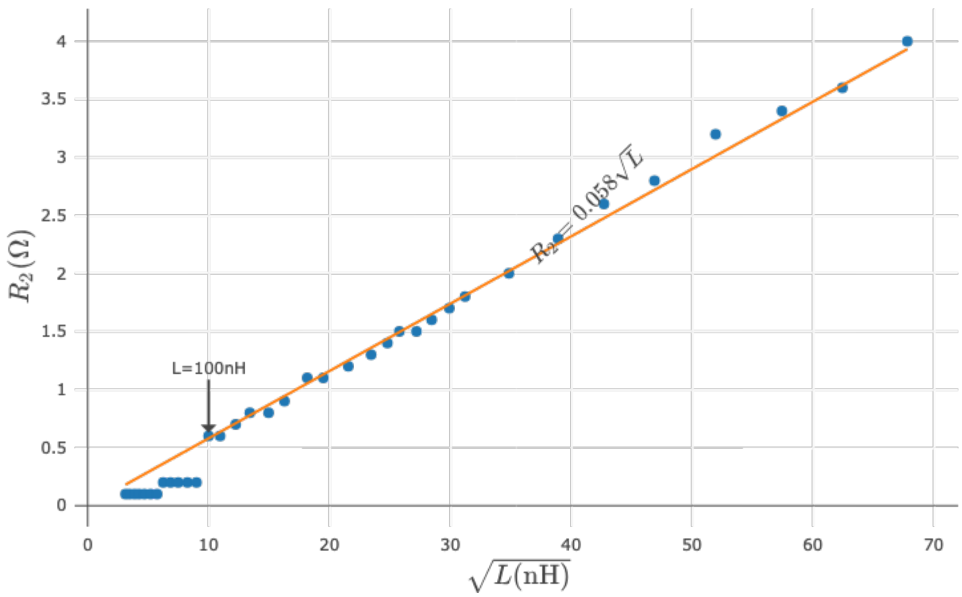

Figure 8: Linear fit to the parameter k vs L.

Practical capacitors

In the zoo of capacitors, things look similar. Figure Figure 9 shows the equivalent circuit for a practical capacitor.

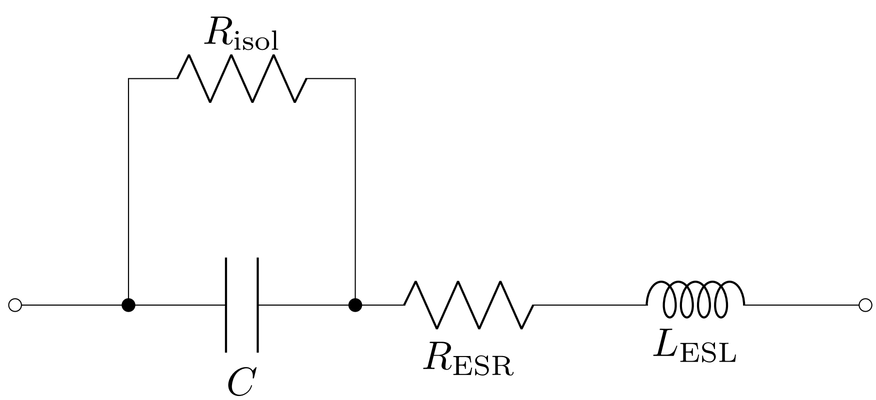

Figure 9: Equivalent circuit for a real capacitor.

TDK has a note on ceramic capacitors as shown in **?@fig-acpESR**.