

Musings on the Weibull distribution

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This is a quick overview of the Weibull distribution with an interactive visualization. We start from simple reliability ideas and build up to the standard Weibull formulas. Along the way, you can play with sliders to see how the shape and scale parameters change the curves and what that means in practice. The goal is to make Weibull feel less mysterious and more like a practical tool.

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Introduction

The Weibull distribution has two parameters: α parameter sets the time scale whereas the shape parameter β sets the shape of the distribution. The failure probability density, cumulative failure distributions and the hazard rate (failure rate) are defined as follows:

$$f_{\alpha,\beta}(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^\beta}, \quad (1)$$

$$F_{\alpha,\beta}(t) = \int_0^t d\tau f_{\alpha,\beta}(\tau) = 1 - e^{-\left(\frac{t}{\alpha}\right)^\beta}, \quad (2)$$

$$h_{\alpha,\beta}(t) = \frac{f_{\alpha,\beta}(t)}{1 - F_{\alpha,\beta}(t)} = \frac{\beta}{\alpha} \left(\frac{t}{\alpha} \right)^{\beta-1}. \quad (3)$$

There is a misconception that Weibull distribution has a theoretical physical basis, which is clearly not true. In fact, Weibull was first to acknowledge that in the very original paper [1]:

... The objection has been stated that this distribution function has no theoretical basis. But in so far as the author understands, there are-with very few exceptions-the same objections against all other [functions], applied to real populations from natural or biological fields, at least in so far as the theoretical basis has anything to do with the population in question. Furthermore, it utterly hopeless to expect a theoretical basis for distribution functions of random variables...

Ernst Hjalmar Waloddi Weibull

I think one of the root causes of this misconception is the way this topic is introduced. Typically, the functions are defined in the order as in Eqs. 1 -3. $F_{\alpha,\beta}$, for example, is so neat and so simple, it almost looks like it comes off of a theoretical derivation. A more natural approach would have started from $h_{\alpha,\beta}$, i.e., the hazard rate. It is the failure rate normalized to the survivors. What can the simplest form of h be? Let's say it just a constant, and see where it gets us:

$$\begin{aligned} h_{\alpha,\beta}(t) &\equiv \lambda = \frac{f_{\alpha,\beta}(t)}{1 - F_{\alpha,\beta}(t)} = \frac{\frac{d}{dt}F_{\alpha,\beta}(t)}{1 - F_{\alpha,\beta}(t)} = -\frac{d}{dt} [\ln(1 - F_{\alpha,\beta}(t))] \\ \Rightarrow F_{\alpha,\beta}(t) &= 1 - e^{-\int_0^t d\tau \lambda} = 1 - e^{-\lambda t}. \end{aligned} \quad (4)$$

There you have it: if you go with the simplest assumption on $h_{\alpha,\beta}$, you get the exponential distribution.

Let's try a bit more generic function for h . Let's say it is a function of t , and try t^κ , where κ is a constant. It also needs a coefficient to fix the units, so let's try the following:

$$\begin{aligned} h_{\alpha,\beta}(t) &\equiv \zeta t^\kappa = -\frac{d}{dt} [\ln(1 - F_{\alpha,\beta}(t))] \equiv \lambda \\ \Rightarrow F_{\alpha,\beta}(t) &= 1 - e^{-\zeta \int_0^t d\tau \tau^\kappa} = 1 - e^{-\frac{\zeta}{\kappa+1} t^{\kappa+1}}. \end{aligned} \quad (5)$$

Again, κ and ζ are some constants, however we do not know what they are. Just to simplify the expression we can redefine $\kappa + 1 \rightarrow \beta$, and $\zeta \rightarrow \frac{\beta}{\alpha^\beta}$, which gives $F_{\alpha,\beta}(t) = 1 - e^{-(\frac{t}{\alpha})^\beta}$.

The take away from this analysis is that, there is nothing mysterious about the Weibull function. It is basically the second simplest guess one can make.

Visualizing failure distributions

A plot of Weibull distributions defined in Eqs. (1) - (3). This is a static copy, find the interactive HTML-document [here](#).

- [1] W. Weibull, “A statistical distribution function of wide applicability,” *Journal of Applied Mechanics*, vol. 18, pp. 293–297, 1951 [Online]. Available: <http://web.cecs.pdx.edu/cgshirl/Documents/Weibull-ASME-Paper-1951.pdf>